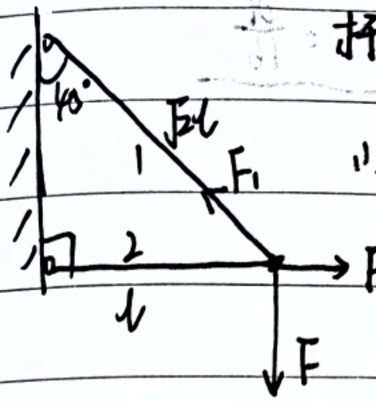
Date:　　　　　　　　　　　　　　　　　　　　　　　　　　Notes

浙江省力学竞赛集训 2025.9

校赛试题（材料力学）

1. 杆件拉伸压缩



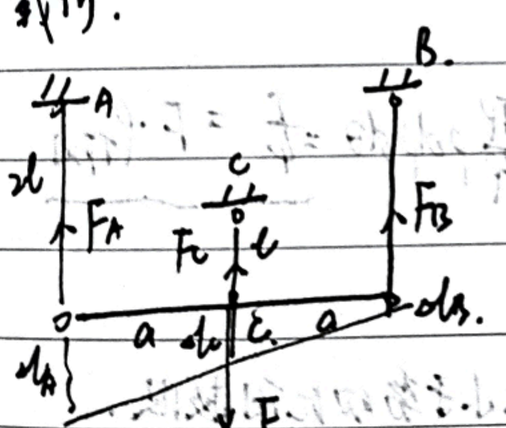
杆横截面为 A，弹性模量 E，杆2长为 l。

(1) 求杆1伸长量

$$\begin{cases}\sum F_x = 0 \\ \sum F_y = 0\end{cases} \rightarrow F_1 = \sqrt{2}F \quad F_2 = F$$

$$\Delta l_1 = \frac{\sqrt{2}Fl}{EA} \quad \Delta l_2 = \frac{-Fl}{EA}$$（受压缩短）

2. 刚性杆，A和B铝绳，直径3mm，弹性模量72GPa，许可应力96MPa；C用钢绳，直径2mm，…200GPa，…124MPa，求许可载荷。



平面任意力系 $\begin{cases}\sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0\end{cases}$（对任意点准

$$\begin{cases}\sum F_y = 0 = F_A + F_B + F_C - F = 0 & ① \\ \sum M_C = 0 = F_A a - F_B a = 0. & ② \\ 2\Delta l_C = \Delta l_A + \Delta l_B \end{cases}$$（变形了仍在同一直线上）

即 $\dfrac{2F_C l}{E_1 A_1} = \dfrac{F_A \cdot 2l}{E_2 A_2} + \dfrac{F_B \cdot 2l}{E_2 A_2}$ ③

解得　　② ④ 对称性：$F_A = F_B$　　拉应力

$F_A = 0.224F$　$F_C = 0.552F$

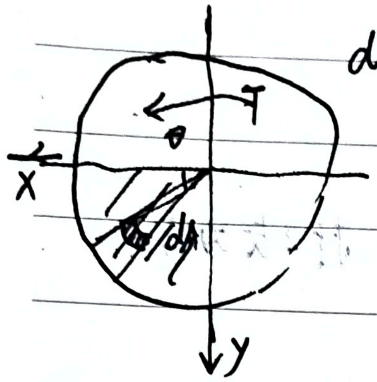
① A/B：$\dfrac{F_1}{A_1} \leq [\sigma_1] = 96MPa$

$F \leq 3033N$

② C：$\dfrac{F_C}{A_2} \leq [\sigma_2] = 124MPa$

$F \leq 705N$

3、圆轴截面扭矩为T，求1/4截面上内力系的合力大小方向、作用点



d

$\tau=\frac{T\rho}{I_\rho}$ $\underline{I_\rho=\frac{\pi d^4}{32}}$ T为力矩（已知）

《指定点应力》

$\tau_{max}=\frac{T\cdot\frac{d}{2}}{I_\rho}=\frac{T}{W_t}$ $\underline{W_t=\frac{d^3}{16}}$

$dA=\rho d\rho d\theta$

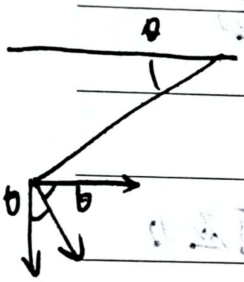
小矩形面积

$dF=\tau\cdot dA$

$=\frac{T\rho}{I_\rho}\rho d\rho d\theta$

$dF_x=dF\cdot\sin\theta$ $dF_y=dF\cos\theta$，

$=\frac{T\rho^2\sin\theta}{I_\rho}d\rho d\theta$



$F_x=\int_0^{\frac{d}{2}}\int_0^{\frac{\pi}{2}}\frac{T\rho^2\sin\theta}{I_\rho}d\rho d\theta=\frac{4T}{3\pi d}$ $(\longrightarrow)$

同理

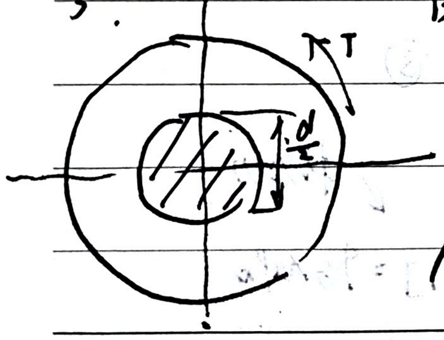
$F_y=\frac{4T}{3\pi d}$ $(\downarrow)$

$F=\frac{4\sqrt{2}T}{3\pi d}$ $(\searrow)$

作用点：$dM_0=\rho dF\Rightarrow M_0=\int_0^{\frac{d}{2}}\int_0^{\frac{\pi}{2}}\rho\frac{T\rho}{I_\rho}\rho d\rho d\theta=\frac{T}{4}=F\cdot\rho_{作用}$

$\rho_{作用}=\frac{3\sqrt{2}\pi d}{32}$

3⁺.



该截面上最大扭转切应力应小于剪切比例极限.

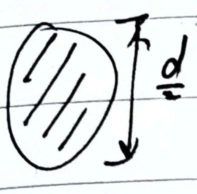
求圆示直径 $\frac{d}{2}$ 的扭矩.

$\Sigma\tau\rho dA\rho=T\rho\Sigma dA$ ← F合力

$M_0=\int_0^{\frac{d}{4}}\frac{T\rho}{\frac{\pi d^4}{32}}2\pi\rho d\rho$.

Date: Notes

3⁺. 解法2.



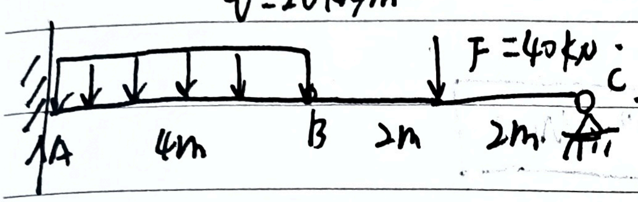
$$\tau_1 = \frac{T_1 \frac{d}{4}}{\frac{\pi}{32}(\frac{d}{2})^4}$$ ← 对于内小截面

$$\tau_2 = \frac{T \frac{d}{2}}{\frac{\pi}{32} d^4}$$ ← 对于大截面

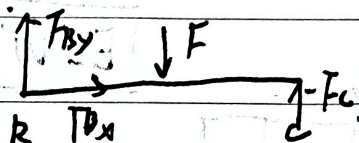
其中 $\tau_1 = \tau_2$,

4.

(1) 画剪力/弯矩图



① 先求约束力

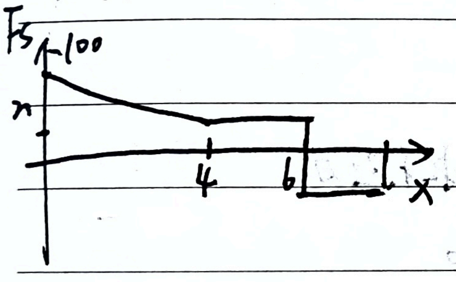


$\Sigma M_B = 0 \Rightarrow F_C = 20kN$

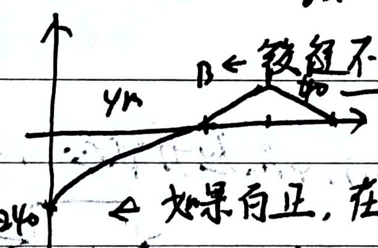
$\Sigma F_y = 0 \Rightarrow F_{Ay} + F_C - F - q \cdot 4 = 0 \Rightarrow F_{Ay} = 100kN$.

$\Sigma M_A = 0 \Rightarrow M_A = -240\ kN\cdot m$ ↻+

② 画剪力图

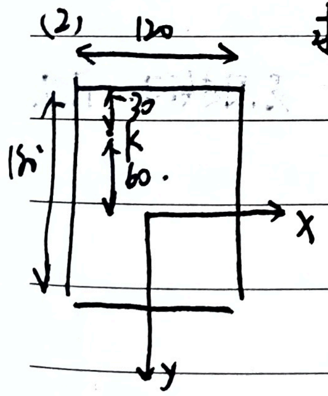


③ 弯矩图 $\frac{dM}{dx} = F_s(x)$



← 如果为正，在C点 M_C 不为零，彰错误

(2) 求A截面处K点正应力与切应力



$M_A = -240\ kN\cdot m$　$F_{Ay} = 100kN = F_{AS}$

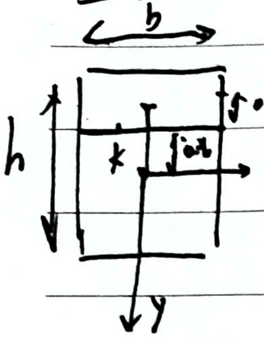
$\sigma = \frac{My}{I_z}$.　$I_z = \frac{1}{12} bh^3$　$(b=120,\ h=180)$

A截面弯矩在下方(负值)，哪一侧受压，A面是下侧(中性面?)受压.

正应力 $\sigma_x = \frac{M_A y}{I_z} = \frac{240\times10^3 \times 0.06}{\frac{1}{12}\cdot 0.12\,(0.18)^3} = 246.9\ MPa$ (受拉)

D材料力学<挠度>

正应力：$\sigma_{max} = \dfrac{M_A (h/2)}{I_z}$　　切应力：$\tau_b = \dfrac{\overset{F_{AS}}{100\times10^3}\,(0.12\cdot0.03\cdot\overset{距离}{0.075})}{\underset{b}{0.12}\times\frac{1}{12}\times\underset{b}{0.12}\times\underset{h^3}{0.18^3}} = 23.1MPa$

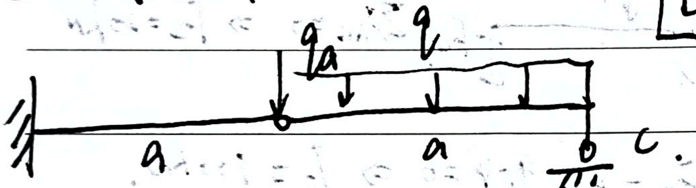


切应力 $\tau_b = \dfrac{F_{AS}\cdot S_z^*}{b\cdot\frac{1}{12}\cdot bh^3}$　(形心中性轴. S_z^*：A面积中心至中性轴

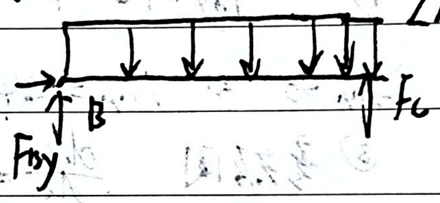
$\tau_{b\,max} = \dfrac{F_{AS}\cdot S_z^*}{b\cdot\frac{1}{12}b\cdot h^3} = \dfrac{3}{2}\cdot\dfrac{F_S}{bh}$ (内力)

取一半面积

5. 求B截面挠度　　$\boxed{EIw''=M(x)}$



① 先求约束力

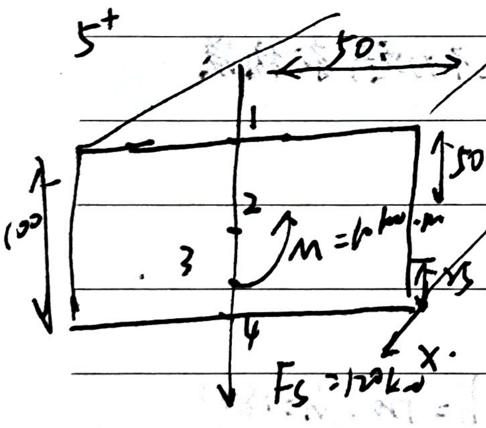


$\sum M_B = 0 \Rightarrow -qa\cdot\frac{1}{2}a + F_C\cdot a$

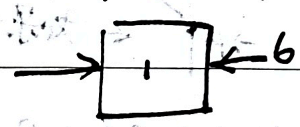
$F_C = \dfrac{qa}{2}$

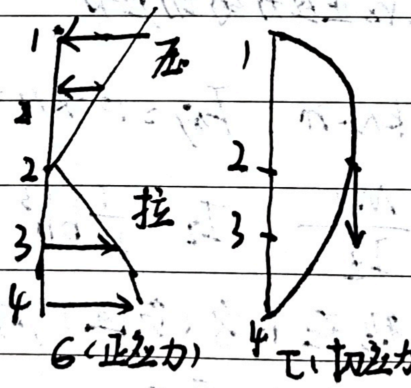
点单元体求应力

5+ 求1,2,3,4点应力状态，并求主应力

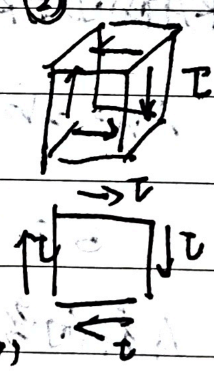


① 对于1





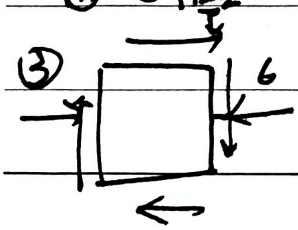
②



τ 共同指向一个边

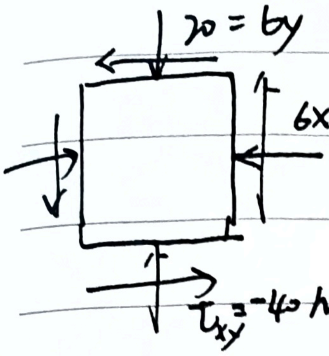
①+②结合

③



④ 同理.

5#. 平面应力分析　(1) 求主应力大小与主平面的方位角



$\tau_{xy} = -40$ MPa. (顺正逆负，对于剪切)

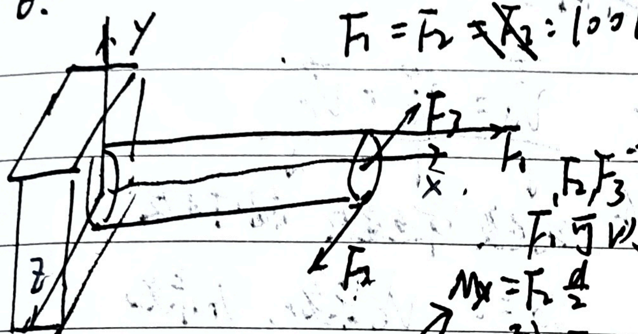
$$\left.\begin{matrix}\sigma_{max}\\ \sigma_{min}\end{matrix}\right\} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

直接代公式

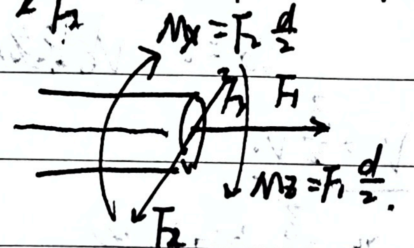
$$tg\,2\alpha_0 = -\frac{2\tau_{xy}}{\sigma_x-\sigma_y} = -4$$

$$\alpha_0 = \begin{cases} -37.98^\circ \\ 52.02^\circ \end{cases}$$

6.　$F_1 = F_2 = \cancel{F_3} = 100$ kN；$F_3 = 90$ kN，求危险截面危险点主应力



F_1 可以平移至中间 → 对于变形分析



F_1：拉、弯曲
F_2：剪、扭转 → 固定端危险
F_3：弯 → 越近越危险 ↓ 固定端

$F_N = F_1 = 100$ kN → σ 正应力

$T = F_2 \frac{d}{2} = 5$ kN·m → τ 切应力(扭转致) ↓ 总弯矩

$M_z = F_1 \frac{d}{2} = 10$ kN·m　　$\sigma_y = 0$；$\sigma_x = \frac{M}{W_z} + F_N/A = 126.6$ MPa.

$M_y = (F_2 - F_3)\frac{d}{2} = 10$ kN·m　　$\tau = \frac{T}{W_t} = 25.5$ MPa.

$M = \sqrt{M_y^2 + M_z^2} = 11.18$ kN·m → 总弯矩

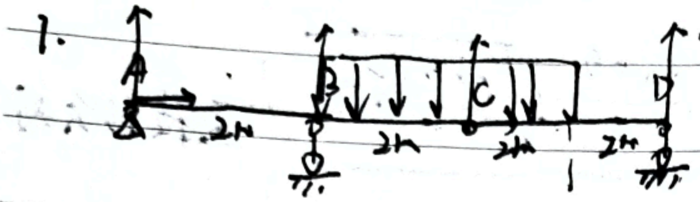
$$\left\{\begin{matrix}\sigma_{max}\\ \sigma_{min}\end{matrix}\right. = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

→ 最大总应力公式

Date: 浙江力竞（校赛）

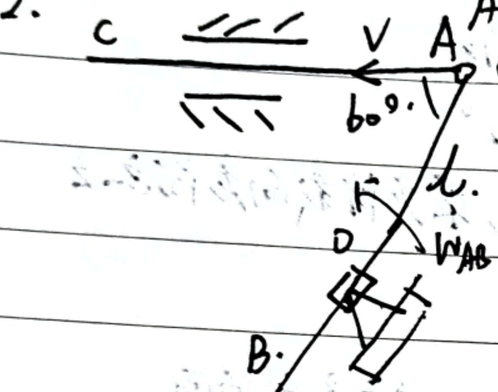
校赛理论力学

1.



$\Sigma M_C = 0$.

2.

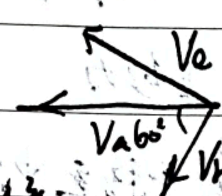


AC均速平动

① 速度

动点：铰链A（AB上）

动系：套筒O.



$$\underbrace{\vec{V_a}}_{\text{绝对速度}} = \underbrace{\vec{V_e}}_{\text{牵连速度}} + \underbrace{\vec{V_r}}_{\text{相对速度}}$$

$V_a = V$　V_e未知，V_r未知

$V_r = \frac{1}{2}V$　$V_e = \frac{\sqrt{3}}{2}V$.

② 加速度

科氏加速度

$$\vec{a_a} = \vec{a_e^t} + \vec{a_e^n} + \vec{a_r} + \vec{a_C}$$

$\vec{a_a}$：绝对加速度 $=0$.

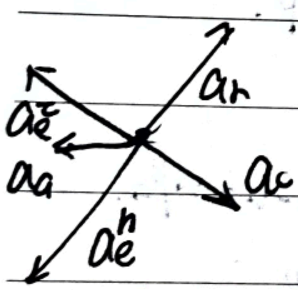
a_e^n：$\omega_{AB} \cdot AO$　　a_C：$2\omega_e V_e$

$\omega_e = \dfrac{\frac{3}{4}v^2}{l}$.

沿 $\vec{a_e^t}$ 方向投影：

$0 = a_e^t - a_C$

$a_e^t = a_C = \frac{3v^2}{4l}$.



a_C的方向：V_r 转90°

（方向由 $V_e \Rightarrow V_r$ 确定）.

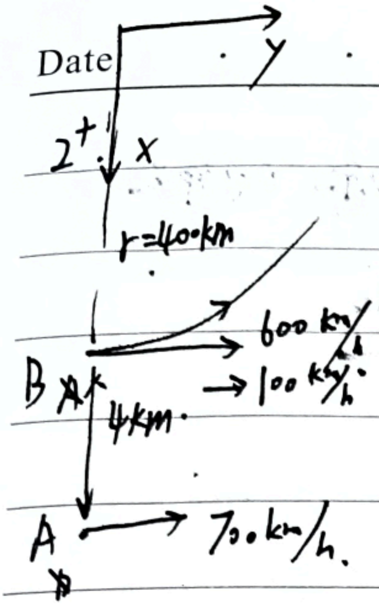
数学（几何）解法：

$x_A = l\cot\varphi$　　$\dot{x}_A = \dfrac{-l\dot{\varphi}}{\sin^2\varphi} = -v$.

$\dot{\varphi} = \frac{v}{l}\sin^2\varphi$.

$\ddot{\varphi} = \frac{v}{l}\sin 2\varphi \cdot \dot{\varphi} = \frac{v^2}{l^2}\sin^2\varphi \cdot \sin 2\varphi$

2+.



求A相对B的速度与相对加速度.

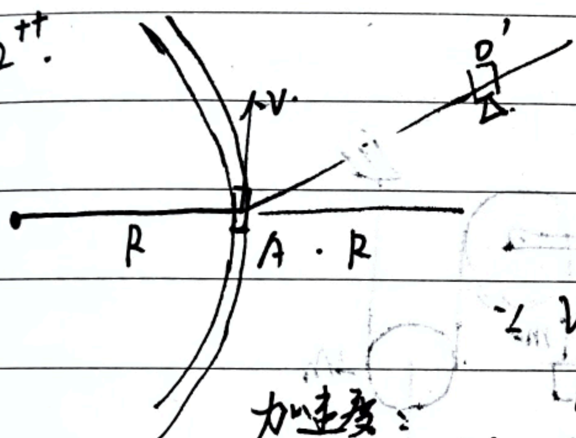
$$\begin{cases} v_{AB}=v_A-v_B=100\,km/h \\ a_{AB}=a_A-a_B \end{cases}$$

$a_B=a_\tau+a_n$ $\quad a_n=\frac{v_B^2}{\rho}$

对于B点 $a_B^n=\frac{v_B^2}{\rho}=a_B^x$

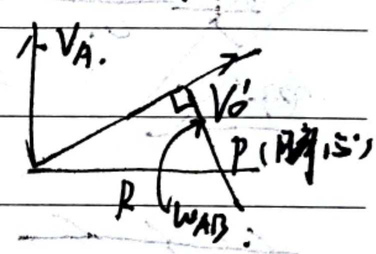
$a_B^y=100\,km/h^2$

2++.



① 动点：O'(AB上)
动系：套筒
静系：地面

$V_e=0$ (牵连速度)



$V_{O'}=V_e+V_r$

$\because V_e=0 \quad \therefore V_{O'}=V_r$ $\quad \omega_{AB}=V_A/2R=V_A/R$

$a_e=0$ $\quad V_{O'}=\frac{1}{2}V$

加速度：$a_{O'}=a_e+a_r+a_c$ (公式)

$=a_A+a_{O'A}$ (平面) $\quad a_{O'A}=a_{O'A}^n+a_{O'A}^\tau$

$a_A=\frac{V^2}{R}$, $\quad a_{O'A}^n=\omega_{AB}^2\cdot O'A$

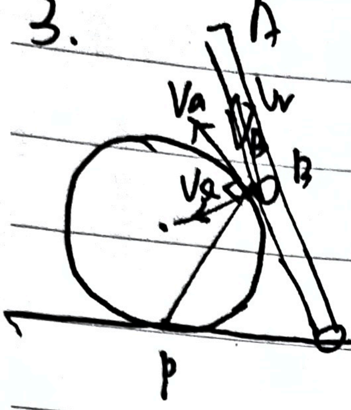
$$\begin{cases} a_{O'}=a_r+a_c \\ a_{O'}=a_A+a_{AO'}^\tau+a_{AO'}^n \end{cases}$$

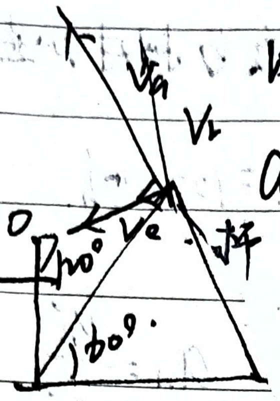
② 另解：

动点：滑块A；动系：套筒；静系：地面

理论力学(核心)

3\.



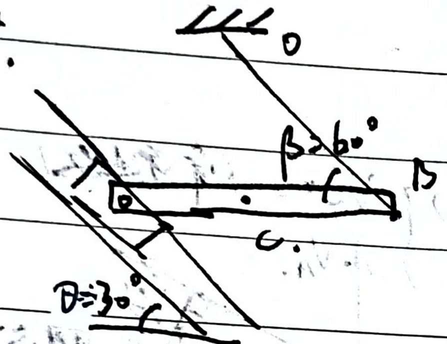


V_r

以B为动点，O_1A为动系，O为静点

$$a_B = a_e^\tau + a_e^n + a_r + a_c$$

4⁺.

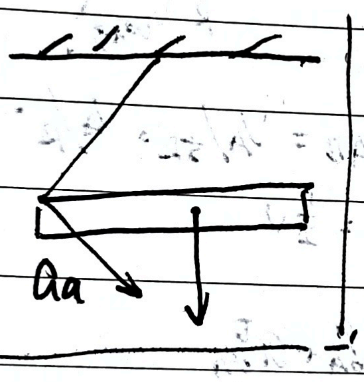


C为刚体质心

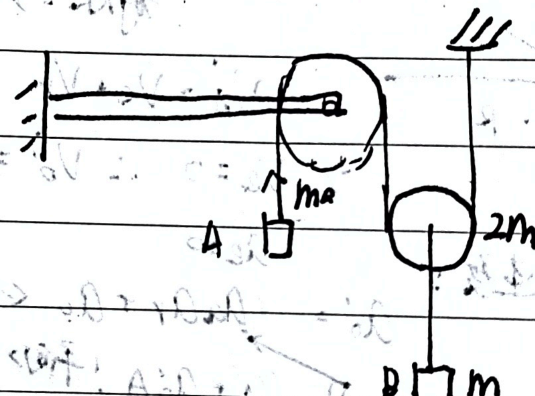
$$\begin{cases} ma_{cx} = \sum F_x \\ ma_{cy} = \sum F_y \\ J_C\alpha_{AB} = \sum m_C(F) \end{cases}$$

$ma = \sum F$ ← 核心公式

4\.

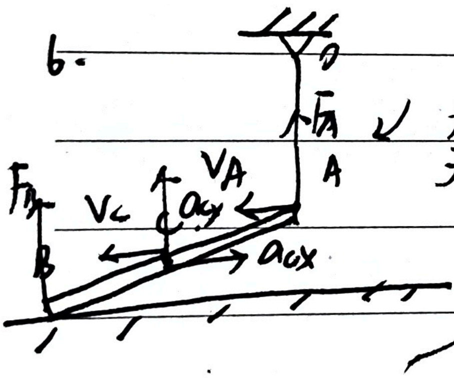


5\.



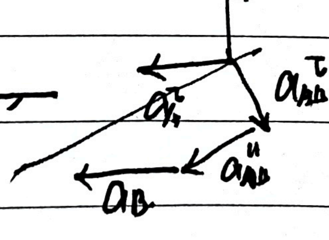
求A上升时加速度 $\sum \frac{1}{2}mv^2 + \sum \frac{1}{2}J\omega^2 = \Delta E$

6\.



$$a_A = a_A^n + a_A^\tau = a_B + a_{AB}^n + a_{AB}^\tau$$

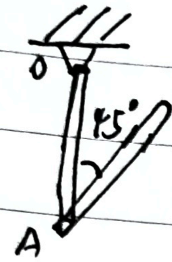
方 大 ✓ ✓ ? ? ✓ ?



D) 浙江力竞

强力练习 10.5.

1.



静止释放的瞬时，两杆的角加速度.

① 运动学分析.（速度与加速度）

设 OA 角加速度 α_{OA}，AB 为 α_{AB}.

+L

I. $a_A = l\alpha_{OA}$. $\xleftarrow{a_A}$

II. 利用加速度的基点法.

即 $a_B = a_A + a_{BA}^{\tau} + a_{BA}^{n}$ 由于初始角速度为0，$a_{BA}^{n} = 0$

$a_{BA}^{\tau} = l\alpha_{AB}$.

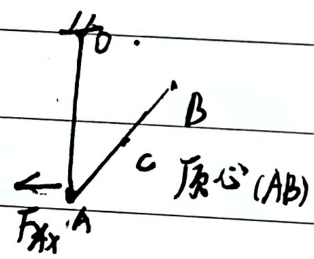
III. B的运动学约束（B点显然没有向水平移动的趋势）.

$\xrightarrow{} +$

$a_{Bx} = 0 = +a_{BA}^{\tau}\cdot\cos45° - a_A$.

$l\alpha_{OA} = a_A = a_{BA}^{\tau}\cdot\cos45° = l\alpha_{AB}\cdot\frac{\sqrt{2}}{2}$

$\alpha_{OA} = \alpha_{AB}\cdot\frac{\sqrt{2}}{2}$



② 动力学方程（微分方程）

I. 杆OA

<定轴转动微分方程> $J_O\alpha_{OA} = \sum M_O(F)$. $J_O = \frac{1}{3}ml^2$（杆的转动惯量）

$\frac{1}{3}ml^2\alpha_{OA} = -F_{Ax}\cdot l$.

II. 杆AB. 以此各个分析

<平面运动微分方程> <基点法> $a_C = a_A + a_{CA}^{\tau}$ $a_{CA}^{\tau} = \frac{l}{2}\alpha_{AB}$.

$$\begin{cases} ma_{Cx} = \sum F_x \\ ma_{Cy} = \sum F_y \\ J_C\alpha_{AB} = \sum M_C(F) \end{cases}$$

$$\begin{cases} a_{Cx} = -a_A + \frac{l}{2}\alpha_{AB}\cos45° \\ a_{Cy} = \frac{l}{2}\alpha_{AB}\sin45° \end{cases}$$

对于 $ma_{cx} = -F_{Ax}$

(3) $m(-a_A + \frac{l}{2}\alpha_{AB}\cos45°) = -F_{Ax}$

对于 $ma_{cy} = mg - F_{Ay}$

$m\cdot\frac{l}{2}\alpha_{AB}\cdot\frac{\sqrt{2}}{2} = mg - F_{Ay}$

得方程组

$$\begin{cases} \alpha_{OA} = \frac{\sqrt{2}}{2}\alpha_{AB} \\ \frac{1}{3}ml^2\alpha_{OA} = -F_{Ax}\cdot l \rightarrow \alpha_{OA} = \frac{-3F_{Ax}\cdot l}{m},\ a_A = \frac{-3F_{Ax}}{m} \\ (3),(4) \end{cases}$$

(3) $+3F_{Ax} + \frac{ml}{2}\alpha_{AB}\frac{\sqrt{2}}{2} = -F_{Ax}$

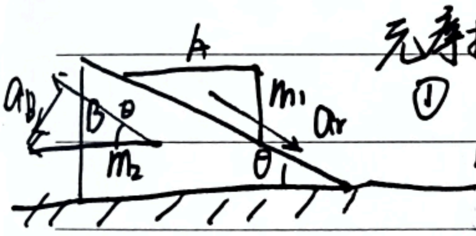
$\frac{\sqrt{2}}{2}\alpha_{AB} = \frac{-8F_{Ax}}{ml} \Rightarrow \alpha_{AB} = \frac{-8\sqrt{2}F_{Ax}}{ml}$

再利用(4)即可得

$\alpha_{OA} = \frac{6g}{11l}$

$\alpha_{AB} = \frac{6\sqrt{2}g}{11l}$

2. 三棱柱运动（起初静止），求B运动的加速度



无摩擦

① 设 $a_B \xleftarrow{+}$，A相对B的相对加速度 $a_r \searrow +$

（基点法）

$a_A = a_B + a_r$

$$\begin{cases} a_{Ax} = a_r\cos\theta - a_B \rightarrow^{+} \\ a_{A,斜面} = a_r - a_B\cos\theta \end{cases}$$

② 对于A进行牛二定律分析

$m_1 g\sin\theta = m_1 a_{A,斜面} \rightarrow g\sin\theta = a_r - a_B\cos\theta \cdots\cdots (1)$

③ 对全系统进行质心运动定理.

C点x坐标

无摩擦力为0：$a_{cx} = 0$.

$$\begin{cases} x_c = \frac{m_1x_A + m_2x_B}{m_1+m_2} \\ x_A = x_B + x_r\cos\theta \end{cases}$$

浙江力竞 <弹力>

质心水平加速度：

$$(m_1+m_2)a_{cx}=0=m_1a_{Ax}+m a_B$$

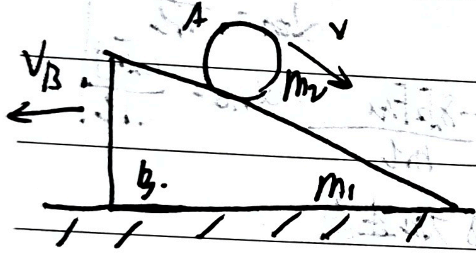
其中 $a_{Ax}=a_r\cos\theta-a_B$.

$$m_1(a_r\cos\theta)=(m_1+m_2)a_B \quad \cdots\cdots (2)$$

$$a_r=g\sin\theta+a_B\cos\theta \quad \cdots\cdots (1).$$

解得 $a_B=\dfrac{mg\sin\theta\cos\theta}{m_2+m_1\sin\theta}$. $+\leftarrow$

3. 平面无摩擦，相对于2从三棱柱换为~~棱~~圆柱，求B加速度



I. 运动学(加速度)分析

设 $a_B=\vec{a}$，A相对B的相对加速度 a_r

$\therefore a_A=a_B+a_r$

II. 质心分析(水平加速度 $a_{cx}=0$).

$$x_c=\frac{m_1x_A+m_2x_B}{m_1+m_2}\rightarrow a_{cx}=0=m_1a_{Ax}+m_2a_B$$

$$0=m_1(a)+m_2(-a+a_r\cos\theta)=0.$$

(1) —— $\therefore m_2a_r\cos\theta=(m_1+m_2)a$

III. 对圆柱使用平面运动微分方程.

① <动量矩定理> $\frac{1}{2}m_2r^2\frac{a_r}{r}=F_fr$ $\leftarrow$ 有摩擦 $J_0=\frac{1}{2}m_2r^2$

<定轴转动微分方程> $F_r=\frac{1}{2}m_2a_r$

$J_0\alpha_f=\sum M_0(F)$ ⊙(F)

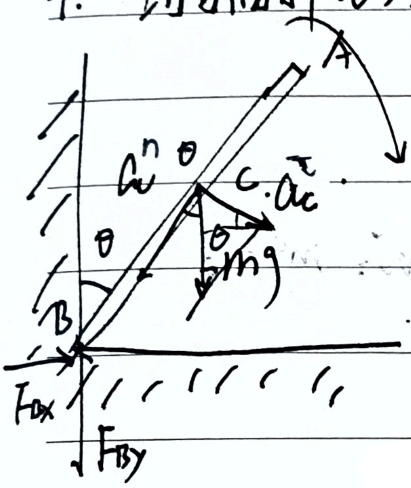
② $m a_r = \sum F_r = m_2 g\sin\theta - F_f$

$= m_2 g\sin\theta - \frac{1}{2} m_2 a_r$

$a_r = \frac{2}{3} g\sin\theta$ ……(2) 再代入(1)

$a = \frac{2m_2 g\sin\theta\cos\theta}{3(m_1+m_2)}$；水平向左.

4. 均匀细杆 l，质量 m，由于微小干扰，杆绕点 B 倾倒



(1) B端未脱离墙壁时杆AB角速度、角加速度

I. 角速度（动能定理）

$\frac{1}{2} J_0\omega^2 = mg\cdot\frac{1}{2}(l - l\cos\theta)$ 其中杆子 $J_0 = \frac{1}{3}ml^2$

$\omega = \sqrt{\frac{3g(1-\cos\theta)}{l}}$

II. 角加速度（$\dot\theta = \frac{d\theta}{dt} = \omega$，$\dot\omega = \alpha$）

$$\left\{\begin{aligned} &\frac{d(\frac{1}{2}J_0\omega^2)}{dt} = \frac{1}{6}ml^2\cdot 2\omega\alpha \\ &\qquad\qquad\qquad \| \\ &\frac{d\,\frac{1}{2}mgl(1-\cos\theta)}{dt} = mg(\frac{l}{2}\sin\theta)\omega \end{aligned}\right\} \Rightarrow \alpha = \frac{3g\sin\theta}{2l}$$

III. F_B 约束力（平面运动微分方程）

$$(1)\left\{\begin{aligned} m_c a_{cx} &= \sum F_{cx} = F_{Bx} \\ m_c a_{cy} &= \sum F_{cy} = mg - F_{By} \end{aligned}\right. \qquad (2)\left\{\begin{aligned} \alpha\cdot\frac{l}{2} &= a_c^\tau = \frac{3}{4}g\sin\theta \\ \omega^2\frac{l}{2} &= a_c^n = \frac{3}{2}g(1-\cos\theta) \end{aligned}\right.$$

$$(3)\left\{\begin{aligned} a_{cx} &= -a_c^n\sin\theta + a_c^\tau\cos\theta \\ a_{cy} &= +a_c^n\cos\theta + a_c^\tau\sin\theta \end{aligned}\right.$$

(1)、(2)、(3) 联立

$$(4)\left\{\begin{aligned} F_{Bx} &= -\frac{3}{2}mg\sin\theta(1-\cos\theta) \\ F_{By} &= mg + \frac{3}{4}mg\sin\theta\cos\theta \end{aligned}\right.$$

D湘江力竞（理力）

(2) 求B脱离墙壁时倾角

$F_{Bx}=0 \Rightarrow a_{cx}=0$

即 $+a_c^n \sin\theta = a_c^t \cos\theta$

$\frac{l}{2}\cos\theta = \frac{l}{2}\omega^2 \sin\theta$

$\boxed{\cos\theta_1 = \frac{2}{3}}$

已知 $\omega=\sqrt{\frac{3g(1-\cos\theta)}{l}}$

$\alpha=\frac{3g\sin\theta}{2l}$

(3) 杆AB着地时（质心速度与角度）

I 质心速度

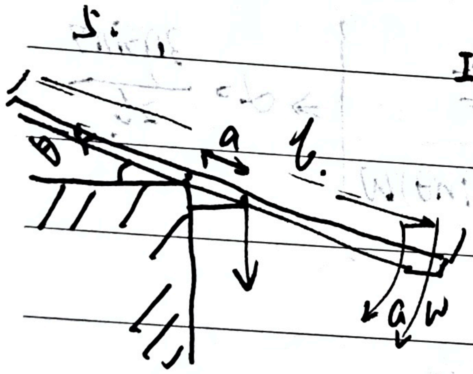
$J_c=\frac{1}{3}m(\frac{1}{2}l)^2=\frac{1}{12}ml^2$

〈动能定理〉

$$\begin{cases} T=\frac{1}{2}mv_c^2+\frac{1}{2}J_c\omega_c^2=\frac{1}{2}mv_c^2+\frac{1}{24}ml^2\omega_c^2=mg\cdot\frac{l}{2} \\ v_c=\frac{l}{2}\omega_c \end{cases}$$

$$\begin{cases} \omega_2=\sqrt{\frac{3g}{l}} \\ v_c=\frac{l}{2}\omega_2=\frac{\sqrt{3gl}}{2} \end{cases}$$

5.



I. 转动惯量（平行轴定理）

$J_c=\frac{1}{3}m(\frac{1}{2}l)^2+ma^2$

II 〈定轴转动微分方程〉

$J_c\alpha=\sum M_O(F)\ (F)$

已知

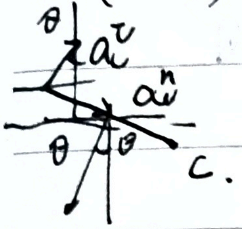
$=mg\cdot a\cos\theta$

$\alpha=\frac{ga\cos\theta}{\frac{1}{12}l^2+a^2}$

Ⅳ. 约束力(质心) <质心的平面运动微分方程>

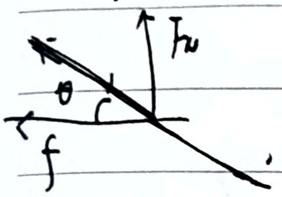
$\begin{cases} m a_{cx} = \sum F_{x} \\ m a_{cy} = \sum F_{y} \end{cases}$ 从已知出发,算 a_c^τ, a_c^n $\omega \to 0$.

$a_c^\tau = a\alpha$ $a_c^n = a\omega^2 = 0$



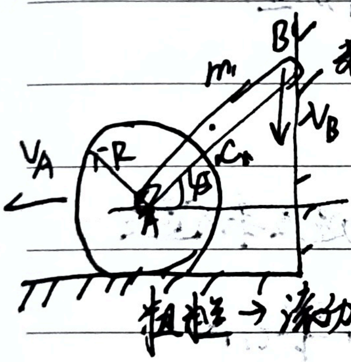
$\therefore a_{cx} = a_c^\tau \sin\theta$

$a_{cy} = a_c^\tau \cos\theta$



$\begin{cases} m a\alpha \sin\theta = F_s = f F_N \\ m \cdot a\alpha \cos\theta = F_N \end{cases}$ 即可得 f.

6. 滚动与滑动 Ⅰ. 运动学<基点法>



$a_B = a_A + a_{AB}^\tau + a_{AB}^n$

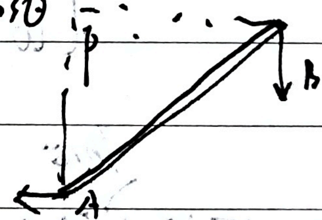
由于初始静止 $a_{AB}^n = 0$.

$a_B = a_A + a_{AB}$ 又已知 $a_{By} = 0$.

再将加速度向y投影

$0 = a_A \sin\theta + a_{BA}^\tau \cos\theta$

即 $a_A = -a_{BA}^\tau$



Ⅱ. 微分动能定理

转动惯量. $J_{AB} = \frac{1}{3} m_1 l^2$ $T_{AB} = \frac{1}{2} J_A \omega_{AB}^2$

$J_{A盘} = \frac{1}{2} m_2 R^2$, $v_A = R\omega_A$

$T_{盘} = \frac{1}{2} m_2 v_A^2 + \frac{1}{4} m_2 (R^2 \omega_A^2) = \frac{3}{4} m_2 v_A^2$

$T_{sum} = \frac{1}{6} m_1 l^2 \omega_{AB}^2 + \frac{3}{4} m_2 v_A^2$

$= \frac{1}{3} m_1 v_A^2 + \frac{3}{4} m_2 v_A^2$

$\frac{dT}{dt} = (\frac{2}{3} m_1 + \frac{3}{2} m_2) v_A a_A$ 左式

Ⅲ. <瞬心法>

若已知刚体上两点的速度方向,

分别作两速度的垂线,即为瞬心

确定瞬心后,杆上各点都是绕

瞬心转动.

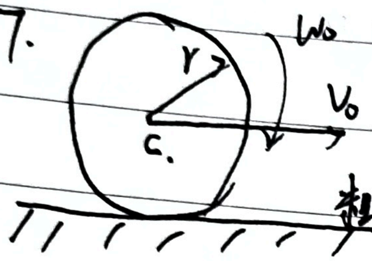
对A, 即可得 $v_A = l\omega_{AB} \cos 45^\circ = \frac{l\omega_{AB}}{\sqrt{2}}$

D)浙江力竞（理力）

$V_C = \frac{1}{2}V_A$　　$P = \frac{dT}{dt} = \frac{1}{2}V_A \cdot m_1 g =$ 左式

解得 $a_A = \frac{3mg}{4m_1+9m_2}$

7.



有初角速度 ω_0，$\omega_0 r < V_0$，动摩擦因数 f，速度 V_0 (1) 求纯滚动所需时间

粗糙 f　　$F_S = fmg$ ←

I. <质心动量定理（平面微分）>

$ma_C = -fmg$　　$a_C = -fg$

II. <质心动量矩定理>　$J_C\alpha = \sum M_C(F)$　$\pm \sum F_n r(F_n)$

$J_C = \frac{1}{2}mr^2$　　$\frac{1}{2}mr^2\alpha = fmgr$

$\alpha = \frac{2fg}{r}$　（←F_S 仅为角加速）

III. 矢量分析与代数分析联立　　条件 $V_C = \omega r$（平动速度等线速度

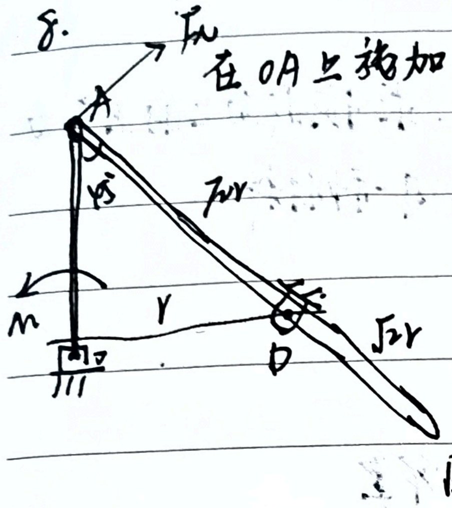
$\begin{cases} V_C = V_0 - fgt \\ \omega = \omega_0 + \frac{2fgt}{r} \end{cases}$　纯滚动：$V_0 - fgt = (\omega_0 + \frac{2fg}{r}t)r$

$V_C = V_0 - fg \cdot t = \frac{2V_0+\omega_0 r}{3}$　←　$t = \frac{V_0-\omega_0 r}{3fg}$

(2) 圆柱体（质心）移动多少距离，开始纯滚动？

匀变速直线运动：$S = V_0 t + \frac{1}{2}a_C t^2$ 或平均速度

8. 在OA上施加M的逆时针力偶，系统初始静止，求AB的角加速度与约束力



I. 定轴转动的杆OA分析

$\langle J_O\alpha = \Sigma M_O(F) \rangle$ $J_O = \frac{1}{3}mr^2$

即 $\frac{1}{3}mr^2 \cdot \alpha_O = M$

$\alpha_O = \frac{3M}{mr^2}$ $a_A = r\alpha_O = \frac{3M}{mr}$

II. 运动学分析（杆AB）

〈基点法〉 $a_C = a_A + a_{CA}^{\tau} + a_{CA}^{n}$ 由于静止 $a_{CA}^{n} = 0$

由于套筒的限制，A 的 $a_{AC}^{\tau} = 0$ 受约束

$\therefore a_C = a_A = \frac{3M}{mr}$ $\alpha_{AB} = 0$

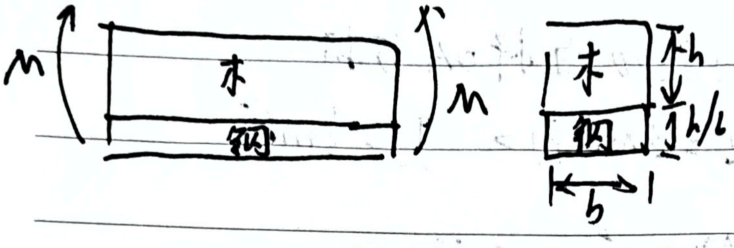
III. 平面运动微分方程（质心运动定理）

$$\begin{cases} ma_C = F_{Ax} + F_N\frac{\sqrt{2}}{2} & \text{（杆的 } x/y \text{ 方向）} \\ 0 = F_{Ay} + F_N\cdot\frac{\sqrt{2}}{2} \Rightarrow F_{Ay} = -F_N\sin 45° \end{cases}$$

联立得：$F_N = \frac{3M(2+\sqrt{2})}{2r}$

材料力学

1、钢木组合梁，木材弹性模量 E_1，钢 E_2，求横截面上的正应力



2. 平面假设（纯弯曲）

常量

（线应变）$\varepsilon = \frac{y}{\rho}$ ，（y 为该点到中性轴距离，ρ 曲率半径）

Ⅱ. 胡克定律计算（应力 = 应变·模量）

$$\begin{cases} 木：\sigma_1 = E_1\varepsilon = E_1\cdot\frac{y}{\rho} \\ 钢：\sigma_2 = E_2\varepsilon = E_2\cdot\frac{y}{\rho} \end{cases}$$

Ⅲ. 静力学平衡（正应力合力为0，只有弯矩）

$\int_A \sigma \, dA = 0.$ （A为面积元） $dA = b\cdot dy$

↓代入

$$\int_{木} E_1\cdot\frac{y}{\rho}\, b\cdot dy + \int_{钢} E_2\cdot\frac{y}{\rho}\cdot b\, dy = 0.$$

↓

$$E_1\int_{木} y\, dA + E_2\int_{钢} y\, dA = 0.$$

Ⅳ. 静力学平衡（弯矩为M）

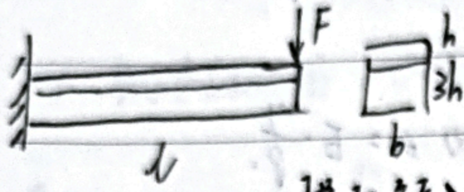
正应力对中性轴的弯矩等于M.

$$\int_A \sigma\cdot y\, dA = M, \Rightarrow \int_{木} E_1\frac{y^2 b}{\rho}\, dy + \int_{钢} E_2\frac{y^2 b}{\rho}\, dy = M$$

D)湘江力竞

3. 悬臂梁的挠度

(1) 若两板完全胶合，求自由端的挠度



<直接按一个梁考虑> $H=4h$.

<横截面>: $I=\frac{bH^3}{12}=\frac{16bh^3}{3}$.

<挠度公式>: $w=\frac{Fl^3}{3EI}$ 悬臂梁挠度公式

$$w_1=\frac{Fl^3}{3E\cdot\frac{16bh^3}{3}}=\frac{Fl^3}{16Ebh^3}$$

(2) 两个板叠放自由端挠度.

↓ <错误做法>

分别计算两块板的挠度，再叠加

上板: $I_1=\frac{bh^3}{12}$ $w_1'=\frac{Fl^3}{3EI_1}$

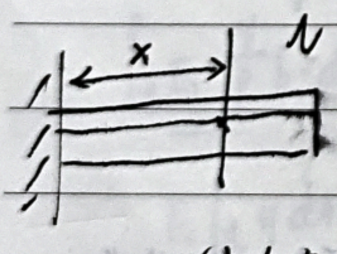
下板: $I_2=\frac{b(3h)^3}{12}$ $w_2'=\frac{Fl^3}{3EI_2}$

$w_2=w_1'+w_2'$.

方法二：分界面ρ相等，←各自弯曲，但不分开

即 $\rho_1+\frac{h}{2}=\rho_2-\frac{3h}{2}$ 由于 $\rho\gg h$ ∴ $\frac{1}{\rho_1}=\frac{1}{\rho_2}$

$\frac{M_1}{EI_1}=\frac{M_2}{EI_2}$ $\frac{M_1}{M_2}=\frac{1}{27}$



$M(x)=-F(l-x)$

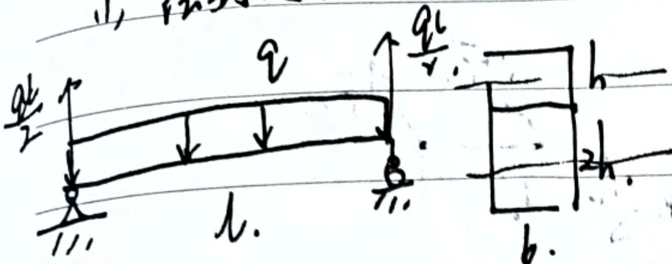
(挠度方程) $M_1=\frac{1}{28}M(x)$

↘ $EI_1w_1''=\frac{F}{28}(l-x)$ → 积两次得 $w_1=w$

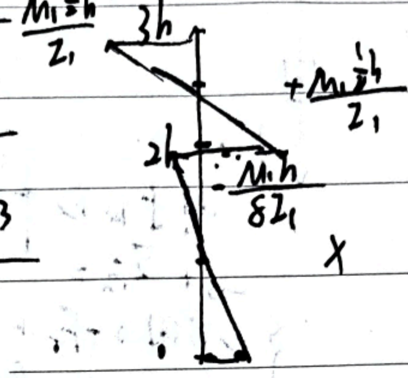
Date:

2. 两个梁相叠，可以自由错动 ($\rho_1=\rho_2$)

1) 绘制两根梁的正应力沿高度示意



← 两梁中性轴都位于形心



上梁：$\sigma_1=\frac{M_1 y_1}{I_1}$　　$I_1=\frac{bh^3}{12}$

下梁：$\sigma_2=\frac{M_2 y_2}{I_2}$　　$I_2=\frac{b(2h)^3}{12}=8I_1$

(2) 叠合面沿轴线方向的错动量（纵向位移差 ↔ 上面拉长，下面变短）.

I. M(x)求：支座反力 $F_{Ay}=\frac{ql}{2}$，均布力在x左的合力 $F=qx$

$$M(x)=\frac{qlx}{2}+qx^2$$

II. M(x)分配为 $M_1(x)$、$M_2(x)$

$$I_{总}=I_1+I_2 \to \boxed{\begin{aligned} M_1(x)&=M(x)\cdot\frac{I_1}{I_{总}}=\frac{1}{9}M(x) \\ M_2(x)&=\frac{8}{9}M(x)\end{aligned}}$$

III. 胡克定律求线应变 → 纵向位移.

$$\varepsilon=\frac{\sigma}{E}=\frac{My}{EI} \leftarrow \sigma=\frac{My}{I}$$

纵向位移：$u(x,y)=\int_0^l \varepsilon\,dx=\int\frac{My}{EI}dx$

上梁：$u_1=\int_0^l\frac{M_1\cdot\frac{h}{2}}{EI_1}=\frac{h}{2EI_1}\int_0^l M_1(x)dx \leftarrow y=\frac{h}{2}$

下梁：$u_2=\int_0^l\frac{M_2\cdot h}{EI_2}=\frac{h}{EI_2}\int_0^l M_2(x)dx \leftarrow y=h$

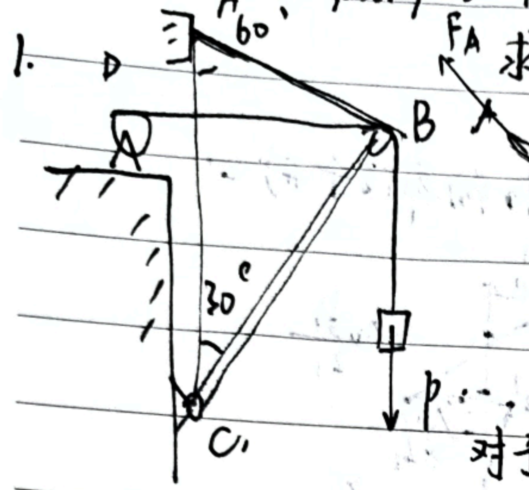
$\Rightarrow \Delta u=u_1-u_2$

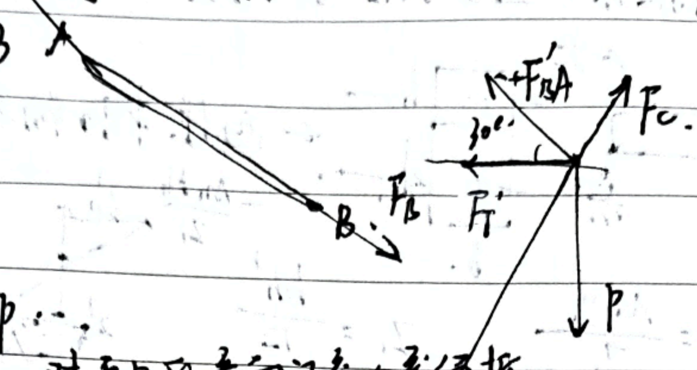
D浙江力竞

理论力学 10.8

9-16. 9-18 练24 11-2

1. 求各个内力





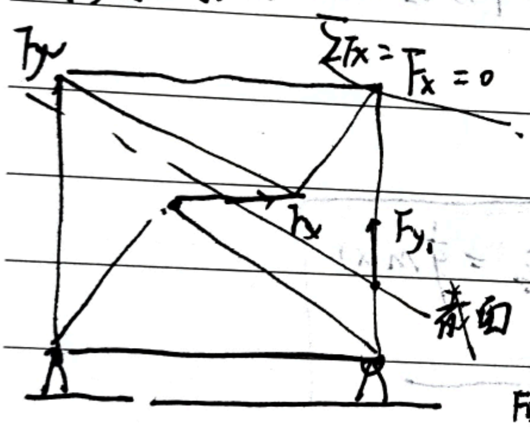
对于B点平面汇交力系分析:

$\xrightarrow{+} \sum F_x =$

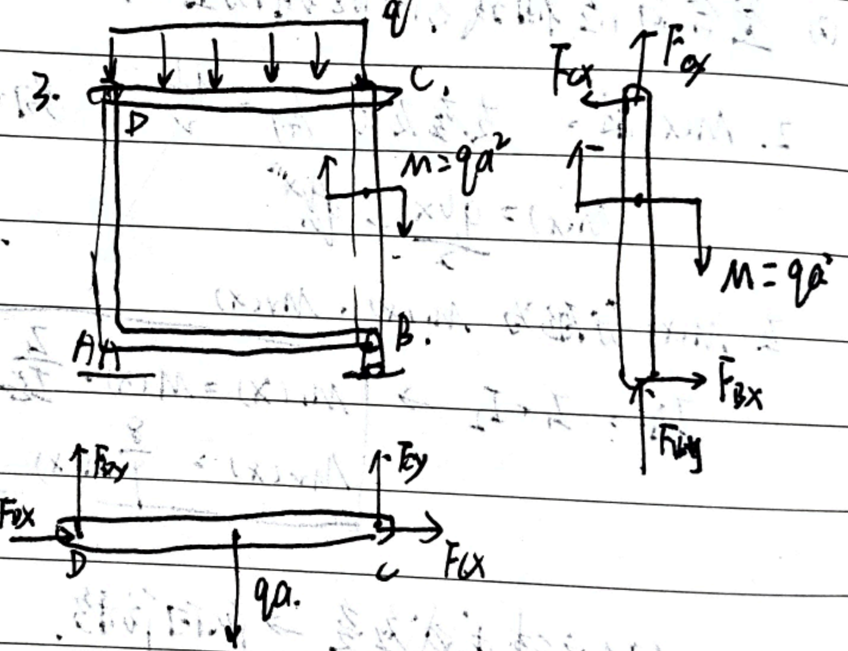
x方向: $-F_T + F'_{BC}\cdot\frac{1}{2} - F'_{BA}\frac{\sqrt{3}}{2} = 0$

y方向: $F_C\cdot\frac{\sqrt{3}}{2} + F_B\cdot\frac{1}{2} - P = 0 = \sum F_y = 0$

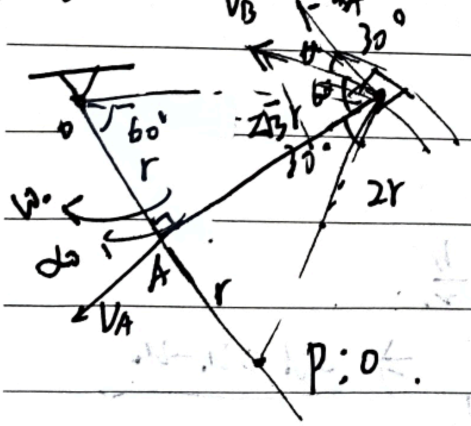
2. 桁架问题

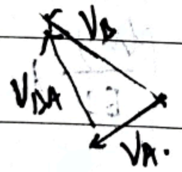


3.



9-18.





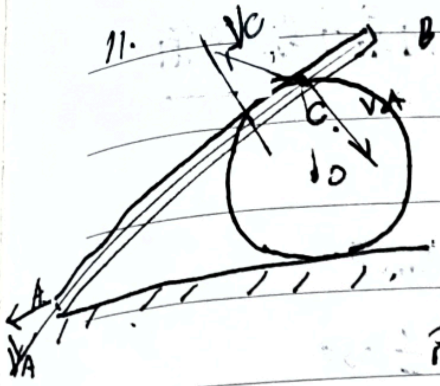
$v_A = \omega_0 r$ $v_B = 2v_A = 2\omega_0 r$

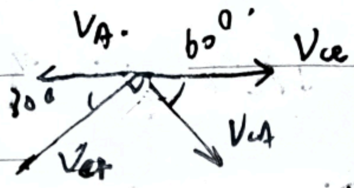
$a_A = \alpha_0 r$

$a_B = a_A + a_{BA}^n + a_{BA}^\tau$

$a_{BA}^n = v_{BA}^2/(2\sqrt{3}r)$

11.





$\vec{V}_C = \vec{V}_A + \vec{V}_{CA}$.

只滚不滑：$\vec{V}_C = V_O + \omega R$.

$V_C = V_A$?

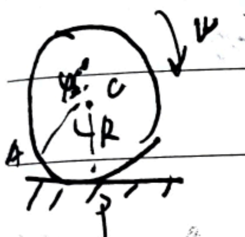
注 $V_C \cos 30^\circ = V_A \cdot \cos 30^\circ$.

$V_C = V_A$ 大小相等.

动量矩计算：$L_C = J_C \cdot \omega + r_C \times m v_C = \frac{1}{2} m v r^2$. （$J_C$: $\frac{1}{2} m r^2$）

$(r_C \times (m v_C))$

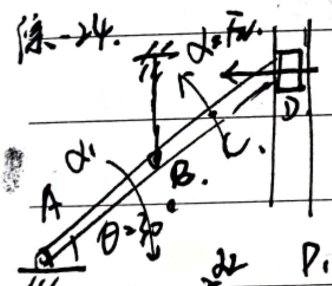
$L_P = r \cdot m \omega r + L_C = \frac{3}{2} m v r^2$



$L_A = m v_C \cdot \frac{\sqrt{2}}{2} R + L_C$.

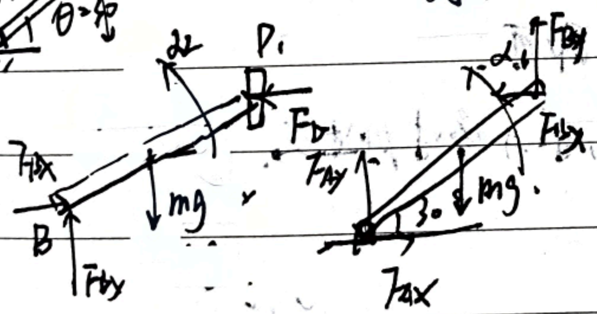
对BD进行平面微分方程分析

练-24.

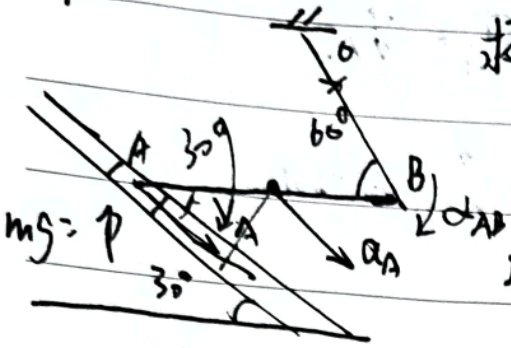


$$\begin{cases} J_{BD} \cdot \alpha_{BD} = -mg \frac{l}{2} \cos 30^\circ + F_D l \sin 30^\circ \\ \Sigma F_y = m a_{yc} = F_{By} - mg \end{cases}$$

对于AB：$J_{AB} \cdot \alpha_{AB} = -mg \cdot \frac{l}{2} \cdot \cos 30^\circ + F_{By} \cdot l \cos 30^\circ$



9.



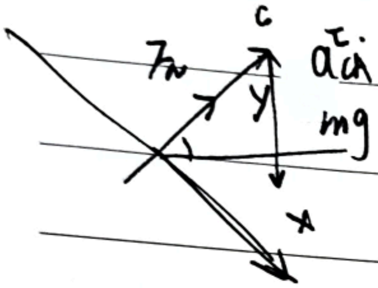
求当剪断细绳瞬间，滑槽约束力与AB角加速度.

$m = P/g$

I. 〈动量矩定理〉 $J_A\alpha = \sum M_O(F)$

$J_A = \frac{1}{3}ml^2$ $\quad \frac{1}{3}ml^2\cdot\alpha_{AB} = mg\frac{l}{2}\cos\theta$

$$\alpha_{AB} = \frac{3\sqrt{3}g}{4l}$$



II. 〈基点法〉运动学分析 F_N.

$a_C = a_A + a^{\tau}_{CA}$ $\quad a^{\tau}_{CA} = \frac{l}{2}\alpha_{AB}$.

将此写为垂直于斜面的x/y方向 $a_{Ay} = 0$

$a^{\tau}_{CAy} = \frac{-l}{2}\alpha_{AB}\cos\theta$.

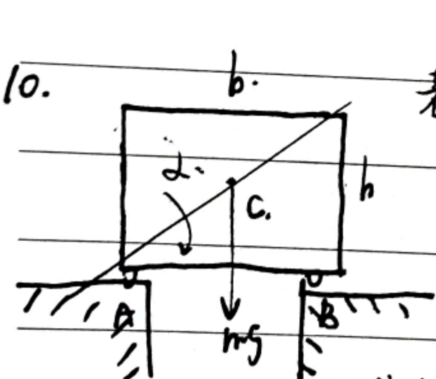
$\underline{a_{Cy}} = \cancel{a_{Ay}} + a^{\tau}_{CAy} = \frac{-l}{2}\alpha_{AB}\cos\theta$

〈质心运动定理〉 $\sum F_y = ma_{Cy}$

$F_N = mg\cos\theta = m\cdot\left(-\frac{l}{2}\alpha\cos\theta\right)$

全部代入即可

10.



若支承B突然移去，求A瞬时加速度.

I. 〈动量矩定理〉

$J_A = J_C + m\cdot\left(\frac{d}{2}\right)^2$ $\quad J_C = \frac{m(b^2+h^2)}{12}$

其中 〈质心平移〉〈平行轴定理〉

$\frac{d}{2}$ 为 $\frac{\sqrt{b^2+h^2}}{2}$ $\quad \Rightarrow \quad \frac{m(b^2+h^2)}{3}$

即 AC距离

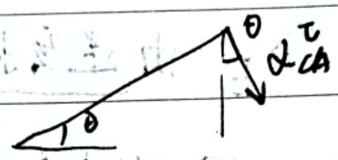
$J_A\alpha_{CA} = mg\cdot\frac{b}{2}$ ∴ $\alpha = \frac{3gb}{2(b^2+h^2)}$

II. 基点法分析 / 投影分析

$a_C = a_A + a_{CA}^{\tau} + a_{CA}^{n}$　$a_{CA}^{n} = 0 \leftarrow v=0$.

$a_C = a_A + \frac{d}{2}\alpha$ → 对光滑平面，应该 $a_{Cx}=0$，$a_A = a_{Ax}$

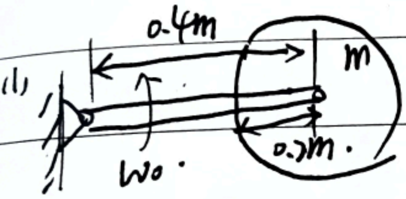
即 $a_{Cx} = a_A - a_{CA}^{\tau}\sin\theta = 0$.



∴ $a_A = \frac{3bgh}{4(b^2+h^2)}$

III. 动量矩计算 $m=25kg$，$R=0.2m$，$\omega_0=\omega_r=4\,rad/s$，OA 轻杆

(1)



计算此情况对O动量矩.　质心C与A重合.

基本概念：刚体对某点O的动量矩

为"①质心动量对O的矩 + ②绕质心的动量矩"

① 质心平动的矩：$L_{O,平动} = r_{OC}\times mv_C$. ← 平动

② 转动矩：$L_{O,转动} = J_C\omega$.

平动：$L_{11} = l\cdot m\omega_0 l = m\omega_0 l^2$　转动：$L_{12} = J_C\omega_A = \frac{1}{2}mr^2\omega_A$

$L_1 = L_{11} + L_{12}$.　牵连角速度与相对角速度同向

(2) $L_{11} = L_{21}$　$L_{22} = J_C\omega_A = \frac{1}{2}mR^2(\omega_0+\omega_r)$

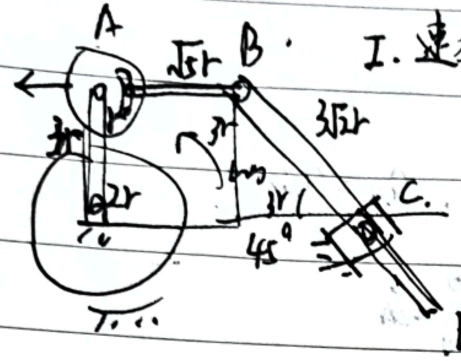
↑ 绝对角速度.

$L_2 = L_{21} + L_{22}$

(3) $L_{11} = L_{31}$　$L_{32} = J_C\omega_A = \frac{1}{2}mR^2(\omega_0-\omega_r) = 0$.

$L_3 = L_{11}$

12. 行星齿轮传动机构 杆BE上与点C相重合一点的速度与加速度



I. 速度分析

$V_B = V_A = 3r\omega_0$ ←

杆BE为平面运动，套筒限制为BE方向。

由速度投影定理

同一刚体两点速度在连线上的投影相等

即 $V_C = V_B \cos 45^\circ = \frac{3\sqrt{2}}{2}\omega_0 r$

II. 基点法加速度分析

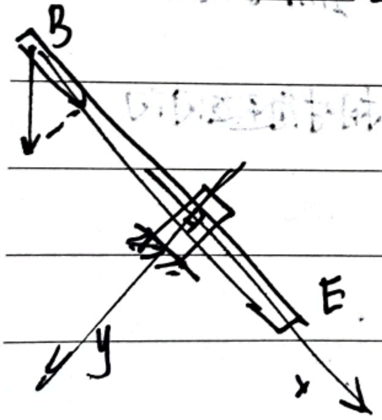
$a_A = a_A^n = OA \cdot \omega_0^2 = 3r\omega_0^2$ ↓

$a_B = a_A + a_{BA}^n + a_{BA}^\tau$ → 由于BA平动 $a_{BA}=0$。

$= 3r\omega_0^2$ ↓

$a_C = a_B + a_{CB}^n + a_{CB}^\tau$ 利用套筒的加速度/速度限制，

即在垂直于套筒方向上，a与v一定为0。



$a_{Cx} = a_{Bx} + \underline{a_{CBx}^n + a_{CBx}^\tau}$

$a_{CBx}^n = a_{CB} = \omega_B^2 BC = \frac{V_B^2 \sin^2 45^\circ}{BC}$

$a_{CBx}^\tau = 0$.

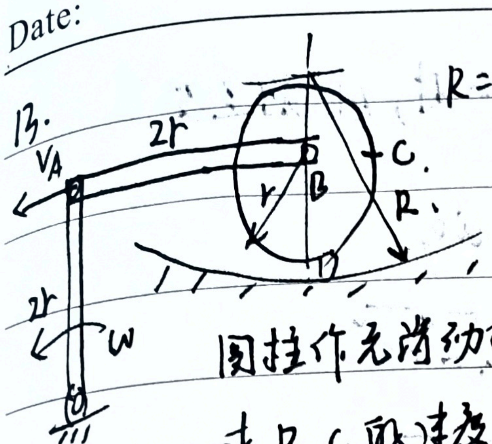
$a_{Cx} = a_{Bx} + a_{CB}^n$

$= \frac{3\sqrt{2}}{2} r\omega_0^2$

$a_{Bx} = a_B \sin 45^\circ$

Date:

13.



① 速度分析

$V_A = V_B = 2r\cdot\omega$（速度投影定理）

$a_B = a_A + a_{BA}^{\tau} + a_{BA}^{n}$.

AB瞬心 $r=\infty$ $a_{BA}^{n} = 0 = a_{BA}^{\tau}$

↓平动

圆柱作无滑动的滚动

求B、C的速度与加速度

《纯滚动情景下瞬心问题》

速度瞬心第二定义：平面运动刚体上，某瞬时速度为0的点。

由于滚动非滑动，接触点D即为瞬心。

对于C点，考虑轮心B的大概运动，与D点瞬心

$\omega_{轮} = \frac{v_B}{r} = 4\ rad/s$

$v_C = DC\cdot\omega_{轮} = 2.83\ m/s$

② 加速度分析.（D瞬心 $a_D \neq 0$, $v_D = 0$ ← 滚动永不滑动）

$a_D = a_B + a_{DB}^{\tau} + a_{DB}^{n} \neq 0$ $a_{DB}^{\tau} = 0$ 水平加速度为0

$a_{DB}^{n} = \omega_{轮}^2\cdot r = 8\ m/s^2$ ↑ $r\alpha_{轮} \rightarrow \alpha_{轮} = 0$

$a_D = -8\ m/s^2$ ↓

$a_C = a_B + a_{CB}^{\tau} + a_{CB}^{n}$

$a_{CB}^{n} = r\cdot\omega_{轮}^2 = 8\ m/s^2$

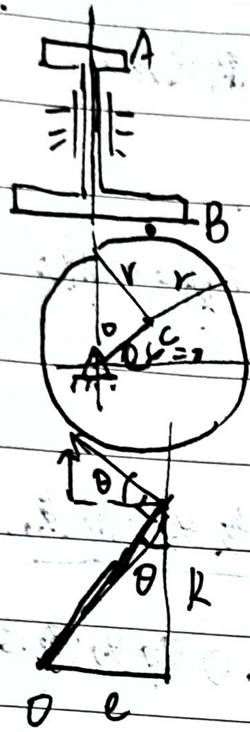
$a_{CB}^{\tau} = 0$

$= 0\ m/s$

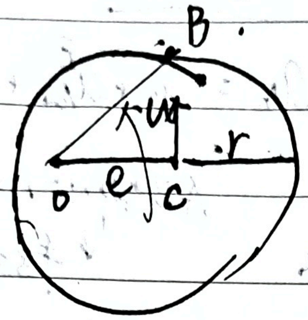
1)浙江力竞.

Notes

14.



平底顶杆凸轮机构，$\varphi=0$时，AB的速度



$v_C=\omega e$

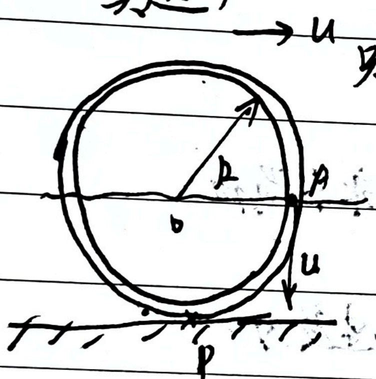
$OB=\sqrt{R^2+e^2}$

$v_B=\sqrt{e^2+R^2}\cdot\omega$

$v_{By}=v_B\cdot\dfrac{e}{\sqrt{R^2+e^2}}$

$=\omega e=v_{AB}$ ✓

15.（真题）



只滚动的细圆环管，小球相对环管的速率为u

(1) 相对加速度：$\sqrt{2}u$.

(2) 对地加速度 $\quad {}''0'' \leftarrow$ 题设

$a_O=0 \quad a_A=a_O+a_A^\tau+a_A^n$

$a_A^n=u^2/R=a_A$

(3) 瞬心 P，$r=\sqrt{2}R$? ×

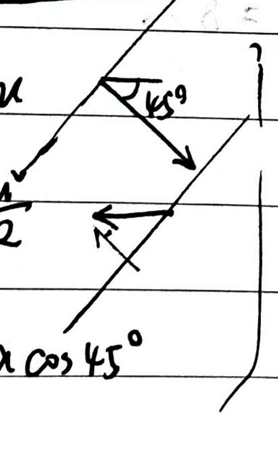
曲率半径的求解.

$\rho=\dfrac{v^2}{a_n}$ a_n为法向分量

$v=\sqrt{2}u$

$a=\dfrac{u^2}{R}$

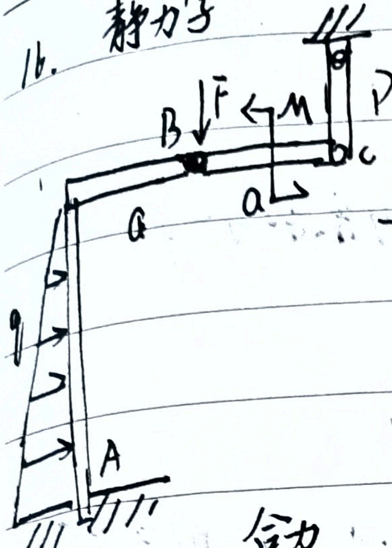
$a_n=a\cos45^\circ$

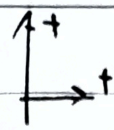


$\rightarrow\rho=2\sqrt{2}R.$

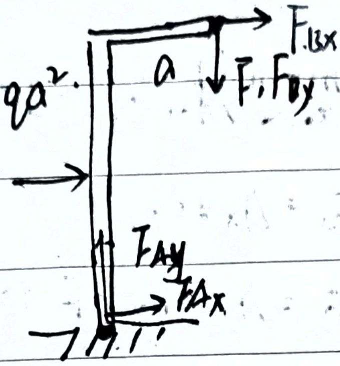
Date:

16. 静力学



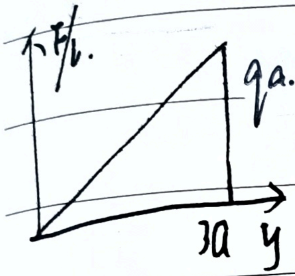


对于AB



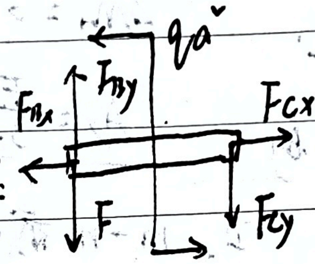
$$\begin{cases} \Sigma F_x = 0 = F_{Bx} + \frac{9}{2}qa^2 + F_{Ax} = 0 \\ \Sigma F_y = 0 = -F_{By} - F + F_{Ay} \\ \Sigma M_A = 0 = \frac{9}{2}qa \cdot \frac{3a}{2} + (F + F_{By})a + F_{Bx} \cdot 3a \end{cases}$$

合力



$\frac{3aq \cdot 3a}{2} = \frac{9}{2}qa^2$

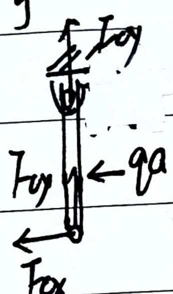
对于BC:



$\Sigma F_x = 0 \Rightarrow F_{Bx} = F_{Cx}$

$\Sigma F_y = 0 \Rightarrow F_{By} = F + F_{Cy}$

对于CD:

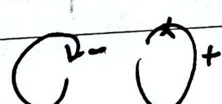


$\Sigma M = aF_{Cx} + qa \cdot \frac{1}{2}a = 0$. $F_{Cx} = -\frac{1}{2}qa = +F_{Bx}$

$\Sigma F_y = F_{Cy} + F_D = 0$

I. 对A进行总分析：(包括F_{Ax}, F_{Ay}, 约束力偶).

$$\begin{cases} \Sigma F_x = 0 = F_{Ax} + 1.5qa - qa \Rightarrow F_{Ax} = -0.5qa \\ \Sigma F_y = 0 = F_{Ay} - F \Rightarrow F_{Ay} = F \\ \Sigma M_A = 0 \Rightarrow M_A - 1.5qa^2 + 2.5qa^2 + qa^2 - 2Fa = 0 \end{cases}$$



$M_A = 2Fa - 2qa^2$

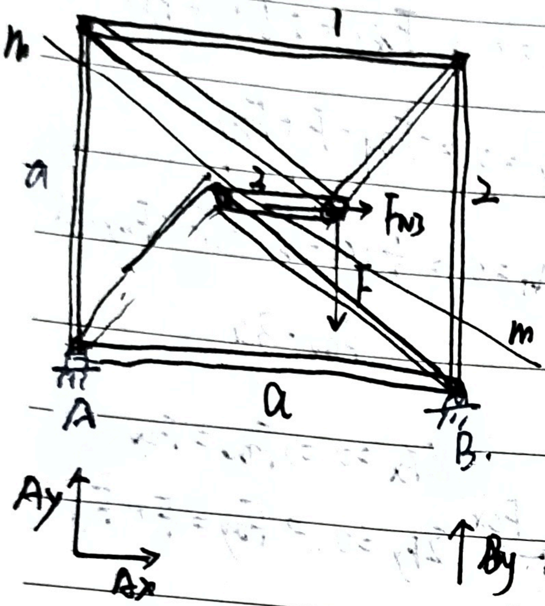
II. 再以AB为研究对象 ← A → 正向

$\Sigma F_x = 0 \Rightarrow -0.5qa + 1.5qa + F_{Bx} = 0$ $F_{Bx} = -qa$

$\Sigma F_y = 0 \Rightarrow F + F_{By} = 0$. $F_{By} = -F$

D 物理力竞

17. 桁架静力学求解



求杆1,2,3的内力

I. 支座反力求解.(汇交力系和为0.)

$\sum F_x = A_x = 0.$

$\sum F_y = A_y + B_y - F = 0.$

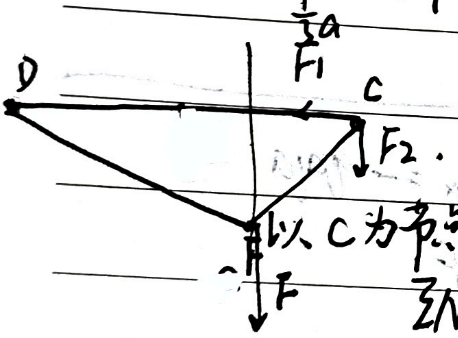
$\sum M_A = 0 = -F\cdot\frac{2}{3}a + B_y\cdot a \Rightarrow B_y = \frac{2F}{3}$

$A_y = \frac{F}{3}$

II. m-m截面取上部

$\sum F_x = 0$, 只有 F_3, $F_3 = 0$.

以D为节点



$\sum M_D = 0 \Rightarrow F\cdot\frac{2}{3}a - F_2\cdot a = 0$

$F_2 = -\frac{2}{3}F$ (压).

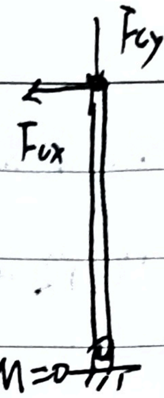
以C为节点的受力

$\sum M_F = 0$, $F_1\cdot\frac{1}{2}a - F_2\cdot\frac{2}{3}a = 0$

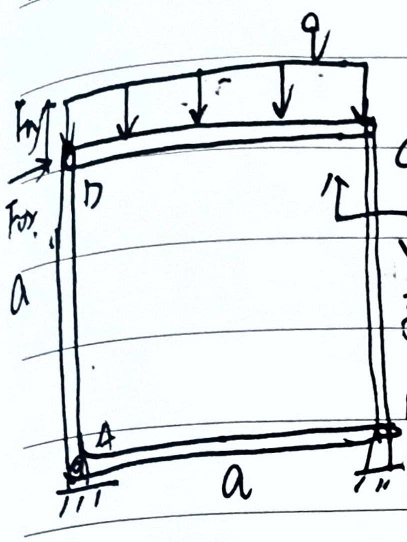
以F作为力矩平衡点 $F_1 = -\frac{4}{9}a$ (压).

Date: Notes

18. 框架静力学求解，求D的受力



由1. 研究杆BC



$M=qa^2$

孤立M，对杆BC分析

$\left\{ \Sigma M_B=0,\ F_{cx}\cdot a-M=0 \right.$

$F_{cx}=qa$ → 对杆CD分析

求约束点，杆端

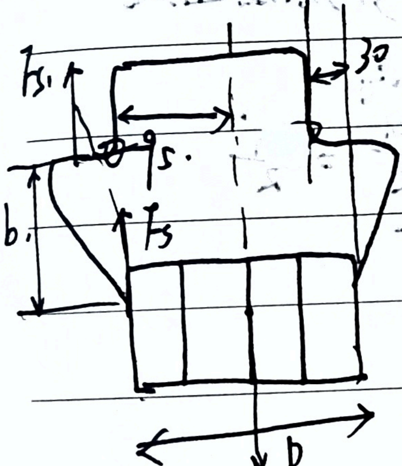
再研究杆CD，

$\left\{ F_{Dx}+F_{cx}=0=\Sigma F_x \right.$ $F_{Dx}=-qa.$

因为分析D，所以对C求矩 → $qa\cdot\frac{a}{2}-F_{Dy}\cdot a=0=\Sigma M_c.$

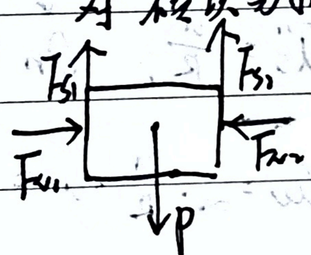
$F_{Dy}=\frac{1}{2}qa.$

19. 摩擦静力学求解. 求b为多大才能把砖夹起



0.25m.

对砖块分析，



$\left\{ \begin{array}{l} \Sigma F_y=0=F_{S1}+F_{S2}=fF_{N1}+fF_{N2} \\ \Sigma F_x=0=F_{N1}-F_{N2}. \end{array} \right.$

联立得：$F_{S1}=F_{S2}=60N$

$F_{N1}=F_{N2}=120N.$

以AGB杆为研究对象，

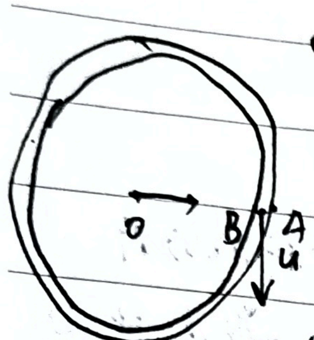
$\Sigma M_G=0 \Rightarrow F\cdot 95mm+F_{S1}\cdot 30mm-F_{N1}\cdot b=0.$

$F\uparrow\ b\uparrow$ 即$F=0$时

$b=110mm$ 即为极限值.

D' 浙江力竞

20. 运动学（十一届真题）



(1) 设D为环管上所置亮落脚点

$V_A = \vec{u_x} + \vec{u_y}$

$V_B = V_A + \vec{u_y} \rightarrow \therefore u_D = \sqrt{5}u$

(2) 亮点相对地面的加速度

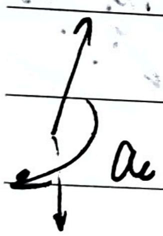
$$a_a = a_e + a_{rn} + a_{r\tau} + a_c + a_n$$

关于 a_c 科氏加速度：由于牵连运动与相对运动相互作用的附加加速度

← 参考系（动系）运动角速度

$\therefore a_c = 2\omega v_r$ ↖ 质点相对转动参考系的速度

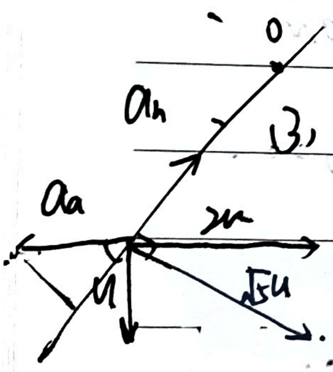
$= 2\omega \times v_r$



$a_e = 0$ $a_{r\tau} = 0$（题设） $a_\tau^n = \frac{u^2}{R}$ 环管A点 $a_n = \frac{u^2}{R}$

$a_c = 2\cdot\frac{u}{R}\cdot u = \frac{2u^2}{R}$

$a_a = \frac{4u^2}{R}$



(3) 曲率半径求解 $a_n = \frac{v^2}{\rho}$

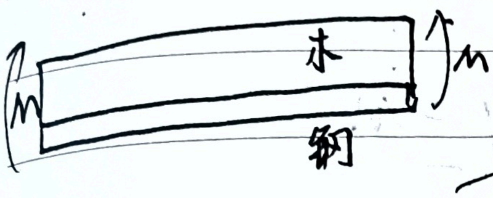
$a_n = \frac{2}{\sqrt{5}}\cdot a_a = \frac{8u^2}{R\sqrt{5}} = \frac{5u^2}{\rho}$

$$\rho = \frac{5\sqrt{5}R}{8}$$

材料力学

1. 正应力计算　　木材弹性模量 E_1，钢材 E_2

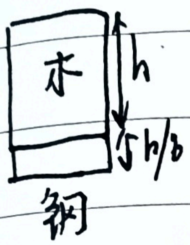
求截面上正应力计算式，给出正应力分布图



~~I. 等效木材计算~~

~~木：$\sigma_w = E_w \cdot \varepsilon$.~~

~~钢：$\sigma_r = E_r \cdot \varepsilon$.~~　　~~$\sigma_r = \frac{E_r}{E_w}\varepsilon$.~~



~~因此将钢材等换为木材~~

~~$b' = \frac{E_r}{E_m} b$.~~

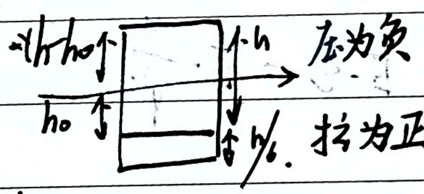
2. 平面假设与应变分布

纯弯曲时的平面假设：$\underbrace{\varepsilon}_{\text{线应变}} = \frac{\overbrace{y}^{\text{到中性轴距离}}}{\rho}$　线应变沿高度线性分布

I. 胡克定律求应力　$\underbrace{\sigma}_{\text{应力}} = E\varepsilon$.

木材：$\sigma_1 = E_1\varepsilon = E_1 \frac{y}{\rho}$　钢材：$\sigma_2 = E_2 \cdot \varepsilon = E_2 \frac{y}{\rho}$

II. 静力学平衡I（正应力合力为0）



$$\int_{木} E_1 \cdot \frac{y}{\rho} \cdot b \cdot dy + \int_{钢} E_2 \cdot \frac{y}{\rho} \cdot b \cdot dy = 0$$

$$E_1 \int_{木} y\,dy + E_2 \int_{钢} y\,dy = 0. \Rightarrow E_1 \int_{-(h-h_0)}^{h_0} y\,dy + E_2 \int_{h_0}^{h_0+h/6} y\,dy = 0.$$

$$h_0 = \frac{3h\left(E_1 - \frac{E_2}{36}\right)}{6E_1 + E_2}$$